

# E - Dynamik in RW spacetime

In[1]:=  $\mathbf{q} = \{\tau, \chi, \theta, \phi\}; \mathbf{dq} = \{d\tau, d\chi, d\theta, d\phi\};$

In[2]:=  $\mathbf{g} = \text{atil}[\tau]^2 \{ \{1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, -\text{Sk}[\chi]^2, 0\}, \{0, 0, 0, -\text{Sk}[\chi]^2 \sin[\theta]^2\} \};$

In[3]:= **MatrixForm** [g]

Out[3]/MatrixForm=

$$\begin{pmatrix} \text{atil}[\tau]^2 & 0 & 0 & 0 \\ 0 & -\text{atil}[\tau]^2 & 0 & 0 \\ 0 & 0 & -\text{atil}[\tau]^2 \text{Sk}[\chi]^2 & 0 \\ 0 & 0 & 0 & -\text{atil}[\tau]^2 \sin[\theta]^2 \text{Sk}[\chi]^2 \end{pmatrix}$$

In[4]:=  $\mathbf{dq} \cdot \mathbf{g} \cdot \mathbf{dq} // \text{Expand}$

Out[4]=  $-d\chi^2 \text{atil}[\tau]^2 + d\tau^2 \text{atil}[\tau]^2 - d\theta^2 \text{atil}[\tau]^2 \text{Sk}[\chi]^2 - d\phi^2 \text{atil}[\tau]^2 \sin[\theta]^2 \text{Sk}[\chi]^2$

In[5]:=  $\mathbf{gcontra} = \text{Inverse}[\mathbf{g}]$

Out[5]=  $\left\{ \left\{ \frac{1}{\text{atil}[\tau]^2}, 0, 0, 0 \right\}, \left\{ 0, -\frac{1}{\text{atil}[\tau]^2}, 0, 0 \right\}, \left\{ 0, 0, -\frac{1}{\text{atil}[\tau]^2 \text{Sk}[\chi]^2}, 0 \right\}, \left\{ 0, 0, 0, -\frac{\text{Csc}[\theta]^2}{\text{atil}[\tau]^2 \text{Sk}[\chi]^2} \right\} \right\}$

In[6]:=  $\text{Detg} = \text{Det}[\mathbf{g}]$

Out[6]=  $-\text{atil}[\tau]^8 \sin[\theta]^2 \text{Sk}[\chi]^4$

In[7]:=  $\text{SqrtDetg} = \text{atil}[\tau]^4 \sin[\theta] \text{Sk}[\chi]^2$

Out[7]=  $\text{atil}[\tau]^4 \sin[\theta] \text{Sk}[\chi]^2$

In[8]:=  $\mathbf{Acov} = \{0, 0, 0, \text{A3}[\tau, \chi]\}$

Out[8]=  $\{0, 0, 0, \text{A3}[\tau, \chi]\}$

In[9]:=  $\mathbf{Acontra} = \mathbf{gcontra} \cdot \mathbf{Acov}$

Out[9]=  $\left\{ 0, 0, 0, -\frac{\text{A3}[\tau, \chi] \text{Csc}[\theta]^2}{\text{atil}[\tau]^2 \text{Sk}[\chi]^2} \right\}$

In[10]:=  $\mathbf{Fcov} = \text{Table}[\text{D}[\text{Acov}[[\text{nu}]], \mathbf{q}[[\text{mu}]]] - \text{D}[\text{Acov}[[\text{mu}]], \mathbf{q}[[\text{nu}]]], \{\text{mu}, 1, 4\}, \{\text{nu}, 1, 4\}]$

Out[10]=  $\left\{ \{0, 0, 0, \text{A3}^{(1,0)}[\tau, \chi]\}, \{0, 0, 0, \text{A3}^{(0,1)}[\tau, \chi]\}, \{0, 0, 0, 0\}, \{-\text{A3}^{(1,0)}[\tau, \chi], -\text{A3}^{(0,1)}[\tau, \chi], 0, 0\} \right\}$

In[11]:=  $\mathbf{Fcontra} = \mathbf{gcontra} \cdot \mathbf{Fcov} \cdot \mathbf{gcontra}$

Out[11]=  $\left\{ \left\{ 0, 0, 0, -\frac{\text{Csc}[\theta]^2 \text{A3}^{(1,0)}[\tau, \chi]}{\text{atil}[\tau]^4 \text{Sk}[\chi]^2} \right\}, \left\{ 0, 0, 0, \frac{\text{Csc}[\theta]^2 \text{A3}^{(0,1)}[\tau, \chi]}{\text{atil}[\tau]^4 \text{Sk}[\chi]^2} \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ \frac{\text{Csc}[\theta]^2 \text{A3}^{(1,0)}[\tau, \chi]}{\text{atil}[\tau]^4 \text{Sk}[\chi]^2}, -\frac{\text{Csc}[\theta]^2 \text{A3}^{(0,1)}[\tau, \chi]}{\text{atil}[\tau]^4 \text{Sk}[\chi]^2}, 0, 0 \right\} \right\}$

In[12]:=  $\text{Sum}[\text{D}[\text{SqrtDetg} \mathbf{Acontra}[[\text{mu}]], \mathbf{q}[[\text{mu}]]], \{\text{mu}, 1, 4\}]$

Out[12]= 0

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In[13]:= jcontra = FullSimplify[Table[1/SqrtDetg
      Sum [D[SqrtDetgFcontra[[mu ]][[nu]], q[[mu ]]], {mu , 1, 4}], {nu, 1, 4}]]
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Out[13]= {0, 0, 0,  $\frac{\text{Csc}[\text{th}]^2 (A3^{(0,2)}[\text{tau}, \text{chi}] - A3^{(2,0)}[\text{tau}, \text{chi}])}{\text{atil}[\text{tau}]^4 \text{Sk}[\text{chi}]^2}$ }
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