The Limits of Prediction: Students' Conceptions of Chaotic Behavior in Nonlinear Systems

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Abstract:

A design experiment utilizing multiple representations of linear and nonlinear phenomena to facilitate students' understandings of classical and modern physics concepts was conducted. Over the course of nine 50-minute instructional periods in January 2006, an enthusiastic, ethnically diverse group of 33 high school students in the San Francisco Bay area participated in a curriculum designed by this author. All students were finishing their first semester of physics with the author as their teacher. Prior to the educational intervention, students were surveyed about their epistemological views on predictability in physics, the limitations of measurement, sensitivity to initial conditions, and holism versus reductionism. Students engaged in multiple inquiry-based investigations in the physics of systems, which included hypothesis generation, experimental design, graphical representations, and using calculators and Boxer-based computer simulations as experimental tools (diSessa, 1995, 2000). Calculators were utilized to employ iteration algorithms demonstrating sensitivity to initial conditions (Burger and Starbird, 2000). After experimenting with simple pendulums, students constructed nonlinear, magnetic chaotic pendulums and used them in experiments investigating the dynamics of chaotic systems. In the final week of the intervention, small groups of students engaged in scaffolded inquiry with Boxer providing a computer representation of the chaotic pendulum. Throughout the curriculum, qualitative data was obtained through student interviews and written responses on worksheets. Post-assessments surveys of students' knowledge of nonlinear dynamics were statistically contrasted with pre-assessments, providing quantitative data to supplement qualitative findings. Overall, there is evidence that the educational intervention helped students understand modern physics concepts in chaos theory and changed students' epistemological beliefs regarding how much is possible to know about a system.

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Introduction

Traditionally, the first semester of high school physics is devoted to elementary classical mechanics. Common physical phenomena such as projectile motion and collisions are modeled algebraically. Approximate, or ideal models of physical systems are made in the process of modeling them mathematically; e.g., small forces are neglected and numbers are rounded. Underlying these approaches is the assumption that small differences in the initial values of a problem's critical variables will not result in drastically different results later in time. Some historians of science have noted that this assumption—that arbitrarily small influences cannot have arbitrarily large effects—was once the "basic idea of Western science" (Gleick, 1987). In the second half of the 20th century, numerous physicists, mathematicians, chemists, and meteorologists working in the realm of nonlinear dynamics proved that this assumption is not true for all physical systems. Systems for which the assumption does not hold were dubbed "chaotic," and can behave unpredictably.

Can high school students understand modern physics concepts like "sensitivity to initial conditions" by engaging in classroom experimentation? Duit and Komorek (1997, 2001) have had success doing so. This research project attempts to duplicate their findings. It also attempts to see if students' views on the degree to which physical systems are predictable can be influenced by an educational intervention. An overarching interest motivating this project was the effect to which traditional physics curricula give students the impression that all systems are predictable. An alternative epistemological position suggested by chaos theory is that some systems are inherently unpredictable because they are infinitely sensitive to their fundamental parameters. Does traditional physics education give students the wrong impression of the epistemology of systems? Answering such a question might involve surveying students before they begin studying physics, and again when they have finished. In this research project, students were not surveyed prior to instruction in physics, but after having completed the first semester of an intermediate level high school physics course. Then, students participated in an educational intervention in which they investigated a chaotic algorithm in mathematics, a chaotic system in the laboratory, and a computer simulation of this system written in the Boxer programming language (diSessa, 2000). Afterward, data from students' post-assessment surveys indicated the extent to which students' views on the nature of the physical world changed by conducting investigations in the physics and mathematics of nonlinear systems.

What is a Chaotic System?

> A chaotic system is defined as one that shows sensitivity to initial conditions. That is, any uncertainty in the initial state of the given system, no matter how small, will lead to rapidly growing errors in any effort to predict the future behavior…In other words, the system is chaotic. Its behavior can be predicted only if the initial conditions are known to an infinite degree of accuracy, which is impossible (Gollub and Solomon, 1996).[1]

"Chaos" derives from the Greek word "□□□□," meaning vast chasm or void. Chaotic systems are dynamical[2] systems that, under specific parameter values, can be unpredictable. Though used extensively in science, the term "system" is difficult to define. In general, a "system" is an ensemble of related elements comprising a whole. Elmer (2002) defines "system" as "a self-contained entity, or an abstract mathematical model for such an entity," and lists the pendulum as an archetypical example. Mathematically, the dynamics of chaotic systems cannot be described using linear differential equations. Prior to computers, nonlinear differential equations were extremely difficult to solve, so physicists preferred to base their theories on linear differential equations. In the late 19th and early 20th centuries, electrodynamics and quantum mechanics were successfully developed using linear differential equations. By the 1950s the physics of microscopic atomic scales was being applied to engineering, but physical phenomenon closer to everyday experience, such as turbulence and fluid dynamics, were still poorly understood. Like the weather and an organism's success in an ecosystem, turbulence and fluid dynamics are nonlinear processes.

In the early 1960s, Edward Lorenz, an MIT meteorologist, used a computer and a simple system of nonlinear equations to model convection in the atmosphere. To his surprise, Lorenz found that systems with only a few variables can display highly complicated, unpredictable behavior. Meteorological outcomes in Lorenz's model were extremely dependent on slight differences in the initial value of one variable, and the model accurately described real-world phenomena. Linear meteorological models quickly became obsolete. In his seminal 1963 paper, Lorenz introduced the phrase "sensitivity to initial conditions" and wrote about how a butterfly flapping its wings in Beijing could theoretically affect the weather thousands of miles away several days later. Thus, sensitivity to initial conditions came to be known as "the Butterfly Effect."

The scientific community came to a consensus that most systems in the real world are nonlinear to some extent, and can exhibit chaotic behavior under certain circumstances. Examples include the weather, generational fluctuations in biological populations, fluid flow, turbulence, mechanical and electrical oscillatory phenomena, heart and brain activity, planetary orbits, economies, and plate tectonics. Sensitivity to

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[1] Dr. Jerry Gollub was one of my physics professors as a Haverford College undergraduate, and was consulted as an advisor to this project. His work is cited in Gleick (1987) and Briggs and Peat (1990).

[2] changing in time

initial conditions can occur without chaos as well: simply multiplying two slightly different small numbers by an extremely large number will result in two divergent products. However, if a system is mathematically "bounded," meaning that its variables stay within a finite range, sensitivity to initial conditions will result in chaotic behavior. Chaotic behavior is also transitive; "transitivity" means that, given enough time, any trajectory in phase space will pass through all points within its bounds with equal probabilities (Nemirovsky, 1993). In thermodynamics, this is known as the "ergodic hypothesis."

Previous Educational Research

Jonassen et al. (1997) states that chaos theory is a scientific perspective that calls into question "many traditional assumptions about learning systems," but his research concerns applying chaos theory to instructional design itself. Ironically, academic journal searches for "chaos" return more articles attempting to apply modern physics concepts to educational research itself than ones that discuss how to teach chaos theory in physics classrooms. The few curricular treatments of chaos theory that have been discussed in educational research literature are documented here.

Ricardo Nemirovsky (1993) had students investigate a Lorenzian Water Wheel, a rather complicated nonlinear system. He found that students expressed several intuitions. One was that periodic regimes are the basic modes of behavior: "If a few irregularities appeared, they dismissed them as exceptions, imperfections, or little mistakes. The students felt that the irregularities had to be explained. It was as if periodic motion was natural and unproblematic, whereas irregularities were puzzling…the students explained that the water wheel was predictable only to the extent that its motion was periodic. When no periodic pattern was discernible, students experienced 'tensions.'" (Nemirovsky, 1993). This result was used to help formulate hypotheses H1, H2, and H3 below.

Another intuition Nemirovsky found was that a big number of affecting variables causes erratic change: "George expressed that something is unpredictable when it is affected by too many variables," like the weather; however, a student named Oscar disagreed: "but the wheel displays both periodic and non-periodic behavior without changing the number of variables" (Nemirovsky, 1993). George's intuition helped form the basis of sub-hypotheses H2f below.

Another intuition was that hidden periodicities underlie all irregularities. Ana and Paul stated that, "a system with a small number of degrees of freedom (perceived as controllable) eventually has to become periodic or predictable" (Nemirovsky, 1993). This intuition helped form the basis of sub-hypotheses H2a below. Other students stated that irregular trajectories reflect discrete randomness (see H2g below). Nemirovsky also found that some students learned that trajectories are determined by the initial conditions (see hypothesis H4 below) and that unstable behavior may be due to being in the border between two regimes in a region of unstable equilibrium.

Adams and Russ (1992) conducted a unit of study for gifted fourth and fifth graders about mathematical periodicity and chaos and the underlying physical processes that produce these phenomena. Hands-on activities, data analysis tools and computer aids were used for instruction in simple periodic motion (as in the pendulum), complex superposition of motions (in vibrations), and chaotic sequences (in stock prices). Their results indicate that young students were able to understand these concepts to a certain extent.

A reason for teaching about modern physics has been identified by Italian researcher Olivia Levrini (2006). She found that high school physics students preferred learning about abstract concepts in quantum mechanics more compared to learning about less abstract, traditional physics concepts. This may be because students are more interested in the philosophical implications of modern physics. Chaos theory certainly has many philosophical implications.

Several articles in *The Physics Teacher* describe attempts to introduce chaotic pendulums into the classroom for teaching about chaos. Cassoro et al. (2004) attached a spark generator to a magnetic pendulum to record evidence of its chaotic trajectory on thermally activated paper. Oliver (1999) recorded students' qualitative explorations of the interaction between gravitational potential energy, magnetic potential energy, and kinetic energy by studying a magnetic pendulum's chaotic behavior.

In 1993, Cornilsen, a student teacher in Germany, embarked on a research project similar to the one described in this thesis. In "The Magnetic Pendulum as a Way to Understand the Basic Idea of Chaos Theory" (a Master's project), Cornilsen indicated that he had students read two pages about chaos theory from a new physics text. Students had to answer questions about it. One week later, they were interviewed about the magnetic pendulum, which was not explained in the text. Results showed that Grade 10 students are able to understand the basics of chaos theory, and opened the door to further research about teaching chaos theory in high school.

German educational researchers Reinders Duit, Michael Komorek, and Jens Wilbers at the Institute for Science Education (IPN), University of Kiel, Germany, have conducted the majority of published research in this field. They state that, "...so far, there appear to be no studies available that address the learners' preinstructional point of view. [Chaos theory] challenges the idea of the deterministic predictability of natural events which is paradigmatic in traditional physics" (Duit & Komorek, 1997). Their findings show how students changed their minds about predictability, and that curriculum teaching chaos theory at the high school level can be successful. [3]

Komorek, Duit, Bucker and Naujack collaborated in a 2001 article about students' "Learning Process Studies in the Field of Fractals," focusing on the question of whether the core ideas of chaos theory and fractals "can be understood by students at the age of 15-17." These researchers note that studies on how students learn about nonlinear systems through new teaching materials (like experiments) are "almost non-existent." The German researchers view their work as "preliminary," but their results encourage them in their attempts to make the core ideas accessible to 15-16 year old students." Students experimented with fractal patterns created through electrolysis. When the experiment was repeated, the same fractal pattern formed. Students were interviewed, and described the pattern as "random." Regarding students' differing conceptions of "random," Komorek, et al. (2001) found one group of students who believed that due to a large number of variables, a system's behavior only appears to be random, but is actually deterministic. They were not surprised when the same pattern repeated. A second group

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[3] See http://www.ipn.uni-kiel.de/abt_physik/nlphys/index_eng.html

believed that random behavior is irregularity, and not determined by principle. However, these students did not question why a similar pattern arose twice.

Researchers at IPN in Kiel use a three-part framework that includes analysis of content structure, empirical investigations, and the continuous reevaluation of the construction of educational models. In accordance with the constructivist epistemological position, they assume there is no 'true' content structure of a particular content area. "What is commonly called the content structure is the consensus of the particular scientific community. Every presentation of the consensus [even in textbooks] is an idiosyncratic reconstruction by the referring author informed by the specific aims the author explicitly or implicitly holds." Texts are analyzed "to reconstruct content structures in such a way that 'elementary' features [key ideas] are emphasized" (Duit & Komorek, 1997). A similar process was followed when the curricular intervention was developed for this research project.

The Patterns Project

> *"A pattern is an identifiable structure with a particular set of relationships that is quite general and surprisingly powerful for explaining and analyzing phenomena in the world."*

This research project has been inspired and guided by the Dr. Andrea A. diSessa's Patterns Research Group at the University of California, Berkeley School of Education. Coming from the background of conceptual change research, the "Patterns of Change and Control" project seeks to identify the basic patterns in physics and use them to teach physics concepts. From the perspective of the Patterns Group, chaos is a pattern, as are oscillation, stability, balancing, equilibration, randomness, threshold, and resonance. These patterns share the qualities of context independence (generality), explanatory power, the ability to be modeled mathematically, and inherent simplicity. Students are given hands-on examples and interactive Boxer-based simulations and their actions and comments are recorded. Analysis of videotaped evidence seeks to identify students' thinking about a pattern and the ways in which they might come to better understand it. Understanding a pattern means seeing its defining features (it essence), identifying what it includes and what it does not (its extension), modeling the patterns' relations, and the pointing out important differences between different embeddings of the same pattern.

Since patterns are context independent, they can be represented in multiple ways, and have " multiple embodiments." DiSessa believes that a beneficial educational task is to give students the opportunity to look at a number of different situations that embody the same concept or pattern. Concrete, specific knowledge of a wide range of situations is necessary for understanding a pattern (diSessa, 2005). By presenting students with several different examples of the same pattern, similarities and differences between multiple embodiments are illuminated, allowing conceptual change to occur.

DiSessa's view contrasts with the dominant view in the field of cognitive psychology: that students presented with several examples of the same general idea are able to overlook the differences and focus only on the similarities between these multiple approaches. In contrast, diSessa argues that, the concrete, situation specific details in each embodiment are extremely important in students' constructions of conceptual ideas. While it is certainly valuable to compare representations, the details of any single presentation of a pattern cannot be overlooked. In practical terms, this means that students must be presented with multiple embeddings in order to truly understand the pattern; one example is not sufficient.

Taking diSessa's view into account means that technology can best be utilized in conjunction with other representational forms, such as a real-world, laboratory-based approach and/or a theoretical, mathematics-based approach. The details specific to each representation should not be swept under the rug; rather, students' understandings are enhanced and conceptual change occurs best when each approach is presented (and constructed in students' minds) as one valid perspective among multiple perspectives on a larger, overarching metaconcept. Since it can be argued that the perspective that students gain from a technologically enhanced educational support will always differ from the alternative perspectives more traditional supports provide, technology has the potential to widen learners' ways of looking at scientific and mathematical ideas, in turn providing a basis for deeper understanding.

In this project, and the patterns project in general, the use of computer simulations is not gratuitous. Simulations are necessary to highlight a key concept in nonlinear dynamics: deterministic unpredictability. By analyzing a real-world chaotic system such as a magnetic pendulum, double pendulum, or chaos bowl, it is fairly easy for students to see that within certain ranges of starting points (initial conditions), that future behavior is unpredictable. However, the fact that these systems are also deterministic is impossible to see without a computer simulation. The reason for this is that chaotic systems can be described by their extreme sensitivity to initial conditions. The degree of sensitivity is infinite. The initial conditions must be exactly the same -- to an infinite number of decimal places -- in order for the system's trajectory (the path of the ball or the pendulum bob) to be the same. In practice, it is impossible to pick two starting points that lie in exactly the same position. Thus, in traditional labs, students never see the same trajectory twice. Without a computer simulation, students may grasp the concept that some systems are unpredictable, yet not see that the system's trajectories are completely determined by initial conditions. Thus, some students may inaccurately conclude that chaotic trajectories are random or based on probability functions, which is not true.

One of the goals of this project was to facilitate in students an understanding that chaotic behavior is unpredictable, yet deterministic. In order for a meaningful prediction to be made, conditions must be known to an infinite degree of precision, which, in practice, is impossible. This concept is both philosophically interesting and fundamental to understanding the nature of chaos.

Participants

This project was devised, organized and conducted by the author when he was a student teacher in a San Francisco Bay Area physics classroom. A cooperating teacher was present, but chose not to participate in this research. The author taught for one period each day in the cooperating teacher’s classroom. During this period, 33 students were instructed, and 33 participated in this research project. 22 students were in 11th

grade, and eleven were in 12th grade. 26 students were male and seven were female. No students had identified special needs or disabilities.

Students' backgrounds and abilities in mathematics and science were in the intermediate range for their school. There were two other physics classes offered at the high school: Conceptual Physics, which is for students weaker in math, and AP physics, which is for students stronger in math. The prerequisites for the intermediate level class of students participating in this research are: (1) having passed chemistry, (2) a C or better in algebra II, or (3) to be currently taking those classes. Although the 33 student participants had experience in algebra, some were still experiencing difficulties with it.

Socially, the students did a satisfactory job working with others. For the group work portions of the educational intervention, groups were assigned based on observations of students' social dynamics. As was the case in class, students had no observed difficulties getting along with each other that were severe enough to interfere with their learning processes. Throughout the project, participants communicated with each other extensively, although conversations were not always on topic.

This research was conducted in one of the most ethnically diverse high schools in the United States. In the project, students groups were all ethnically heterogeneous. From surveys given on the first day of class, four students identified Vietnamese as a language spoken in the home. One listed Cambodian, five listed Tagalog, three listed Chinese, two listed Spanish, and one listed Arabic as languages spoken in the home. Students' ethnicities were never officially documented, but three students appeared to be of mostly European descent, one appeared to be East Indian, one appeared to be Latino, and one appeared to be African American. The most common ethnicity was Filipino. Several students were of mixed ethnicities, and several were first generation Americans.

All students were designated to be proficient English speakers, but 17 students (52%) indicated on a first day of class survey that in the home they speak a language other than English, or in addition to English. Some students did not answer the question, so the actual number may be higher. Although none were officially recognized as ELL or ESL students, some students had major difficulties writing and speaking English. No linguistic resources or information about students' backgrounds were provided to the teacher.

Setting

The community in which the research was conducted is one of the most ethnically diverse in the United States. Overall, the community's support for education is high. There are a large number of immigrants in the community, but the community's diversity is not completely due to recent immigration; many families of many different ethnicities having been living in the area for several generations. The community is not as poor as others in the San Francisco Bay area, but pockets of poverty, linguistic isolation, and low educational attainment rates mark it. In its largest census tract, 54% of the adults over the age of 25 do not have a high school degree, and 30% of these did not reach the ninth grade.

The high school in which this research was conducted is one of the largest in the United States. It has over 4000 students, and its "population status" is the "Urban Fringe of a Large City," according to the California Department of Education. Almost 80% of its students are of color, with over 20% from immigrant families. There are 1091 students in 9^{th} grade, 1053 in 10^{th} grade, 1132 in 11^{th} grade, and 973 in 12^{th} grade. 769 classes are held at the school, and the average class size is 31.0. There are 1383 computers at the school and an average of 3.2 students per computer. 159 classrooms have Internet access. In the 2004/2005 school year, the school's API base was 692, and its statewide rank was 6, slightly above average. Similar schools ranked 3. In 2004/2005, 12% of the school's students were African American, 23% were Asian, 19% were Filipino, 26% were Latino, 2% were Pacific Islander, and 17% were white (not of Hispanic origin). 25% of students participated in the free or reduced price lunch program, NSLP. In the physics classroom in which this research was conducted, there were six functioning computers.

The author began teaching non-calculus based physics to the 33 student participants on the first day of the 2005-06 school year in early September, 2005. The research was conducted in January, 2006. Prior to beginning the research, the author had covered chapters 1 through 11, or pages 1 through 240, in <u>Merrill Physics: Principles and Problems</u> by Paul W. Zitzewitz, et. al. From September 2005 to January 2006, the following topics were taught: "What is Physics?," "A Mathematical Toolkit," "Describing Motion: Velocity," "Acceleration," "Forces," "Vectors," "Motion in Two Dimensions," "Universal Gravitation," "Momentum and Its Conservation," "Work," and "Energy." No topics outside of elementary mechanics were covered in detail.

Goals

In chronological order, the goals of this research project were to:

1. Develop and implement a curriculum to teach modern physics concepts in chaos theory, complexity theory, and nonlinear systems theory to urban high school students.
2. Assess students' epistemological and conceptual views regarding the nature of physical systems before, during, and after the curricular intervention.
3. Analyze data for evidence of students understanding the modern physics concepts to discover how the curricular intervention helped students learn them.
4. Analyze data for evidence indicating that students' epistemological and conceptual views regarding the nature of physical systems changed.

Goal 1 was completed prior to the teaching event described in the procedure below. In goals 2 and 4, "epistemological" refers to students' views on what it is possible to know about a physical system; in other words, acquire information about the specificities of what cannot be known about the system.

Goal 3 was accomplished through reviews of both videotapes of students' words and actions during the intervention and copies of student-completed instructional handouts. It was assumed that the structure of the intervention and the instructor's comments were together responsible for students' gains in understanding the modern physics concepts. The degree to which the actions of the primary investigator / teacher / author of this project influenced students' understanding was deemed too difficult to distinguish, especially given the bias inherent in the instructor also being the writer of the curriculum. Verbatim fragments of dialog and students' written comments were reviewed and quoted in the curriculum description section of this thesis. Goal 3 provided evidence for all hypotheses and sub-hypotheses.

Goal 4, in conjunction with Goal 2, provided evidence for hypotheses H1 and H2, including sub-hypotheses H2a, H2b, H2c, H2d, H2d, H2e, H2f, and H2g (see below).

Procedure

Prior to the intervention, each student filled out a two-page pre-assessment survey (see Appendix C). Two different versions of each page were produced, with questions worded and ordered differently. This was done in order to diagnose the effects of question wording on student responses. For example, statements worded using the word "not" in one version were worded without the "not" in the other version. This way, the extent to which students answered questions affirmatively could be documented.

The two versions of the first page of pre-assessment questions are hereafter referred to as A and B, and are followed by the question number. The two versions of the second page are C and D. Approximately one quarter of the class received pages A and C, another quarter A and D, a third, B and C, and a fourth received versions B and D.

The educational intervention took place over a span of eight 50-minute instructional periods from January 11-23, 2006. On the first day, the class as a whole performed a dueling calculators activity that was modified from an activity described in the innovative instructional text The Heart of Mathematics (Burger and Starbird, 2000). The class was divided in two. Half entered in 0.510 as an initial seed value into their calculators. The other half entered a number less than but as close to 0.510 as the resolutions of their calculators would allow. Students then performed an iterative algorithm, multiplying by 180 and hitting the sine function key. Students saw that as the number of iterations increased, values began to diverge. However, students with the same make and model calculator had the exactly the same results if they started with the same seed value.

On the second day, a quarter of the class investigated stable and unstable equilibrium states using bowls and marbles. Another quarter observed chaotic turbulence in a water faucet (Gleick, 1987; Briggs and Peat, 1989). These activities were scaffolded as inquiry investigations, but students did not appear to learn very much from them. The remaining half of the class worked on problems unrelated to this intervention, and never performed the "faucet" and "bowl" activities.

On the third day, half of the class began investigating the energy dynamics in a simple pendulum, a tie-in with prior class content. They drew plots of the pendulum's swing in phase space. Then, they performed inquiry investigations of the magnetic pendulum, writing hypotheses before conducting experiments. One experiment involved seeing if the pendulum bob followed the same trajectory if released twice from approximately the same point in space. At the end of the period, students were encouraged to design and perform their own experiments, but few did so. On day four, the other half of the class followed the same curriculum.

On the fifth and sixth days, groups of eight students (a quarter of the class) observed computer generated plots of a chaotic pendulum in phase space. They then experimented with a Boxer simulation of the magnetic pendulum, first drawing hypotheses of what a map matching the starting and final positions of the pendulum bob would look like. The Boxer program was used to obtain simulated data.

On the seventh and eighth days, a real magnetic pendulum was displayed to groups of eight, and students devised explanations of its behavior. They then used the Boxer program as in days five and six.

On January 24, 2006, students filled out post-assessment surveys. Two versions of this instrument were distributed: E and F. Just as in the pre-assessments, questions were worded and ordered differently in E and F in order to diagnose the effects of slight variations, such as including a negative modifier.

Date	Students	Description of Activities
January 10	n=27	pre-assessments completed
January 11	n=28	dueling calculators activity
January 12	n=16 Group 1: QP, AP, MR, JM Group 2: KC, DS, PA, FR Group 3: WL, DN, BT, AC Group 4: MS, MV, MP, EZ	half of each group observed turbulence in a water faucet, the other half investigated stable and unstable equilibrium states using bowls and marbles
January 13	n=16 Group 1: QP, AP, MR, JM Group 2: KC, DS, PA, FR Group 3: WL, DN, BT, AC Group 4: MS, MV, MP, EZ	-mechanical energy in a simple pendulum -phase space plots of simple pendulum -experimentation with magnetic pendulum: hypothesis1, observation1, hypothesis2, observation2, hypothesis3, observation3
January 17	n=16 Group 5: CC, TL, IC, SS Group 6: CP, JCh ,MT, IP Group 7: JG, CS, BS, MSt Group 8: AB, JQ, ED, JC	-mechanical energy in a simple pendulum -phase space plots of simple pendulum -experimentation with magnetic pendulum: hypothesis1, observation1, hypothesis2, observation2, hypothesis3, observation3
January 18	n=8 Group 2: KC, PA, FR, EZ Group 6: CP, JCh ,MT, IP	-observation of computer generated phase space plots of a chaotic pendulum -experimentation with Boxer simulation of magnetic pendulum
January 19	n=8 Group 3: WL, DN, BT, AC Group 7: JG, CS, BS, MSt	-observation of computer generated phase space plots of a chaotic pendulum -experimentation with Boxer simulation of magnetic pendulum
January 20	n=8 Group 4: MS, MV, MP, DS Group 8: AB, JQ, ED, JC	-observation of real world magnetic pendulum -experimentation with Boxer simulation of magnetic pendulum
January 23	n=8 Group 1: QP, AP, MR, JM Group 5: CC, TL, IC, SS	-observation of real world magnetic pendulum -experimentation with Boxer simulation of magnetic pendulum
January 24	n=30	post-assessments completed

Table 1. This chart outlines the curricular intervention.

Hypotheses

Hypothesis H1, "prior predictability": After learning traditional high school level classical mechanics and prior to the modern physics educational intervention, most students believe that all physical systems are predictable.

It is possible that the view of the nature of physics that students get from traditional curricula implicitly implies or explicitly indicates that all phenomena can be numerically analyzed and all future events can be predicted, given the right measuring devices.

Hypothesis H2: During the educational intervention, students will learn modern physics concepts about the nature of chaotic physical systems and change their beliefs about the epistemology of physical systems:

Sub-hypothesis H2a, "limited of prediction": Students will learn that in some systems (chaotic systems), there are limits to what it is possible to know about the system's future behavior, since no measurement of the system's parameters can be infinitely precise. In other words, some systems are not predictable.

Sub-hypothesis H2b, "modeling uncertainty": After the intervention, students will have a greater tendency to disagree with the view that given the right measuring devices, all systems can be modeled in a way that allows their futures to be predicted.

Traditional high school physics instruction is based around the measurements of linear systems, or linear models of real world systems. Such instruction may give students the impression that all systems' futures can be predicted using models. No model can predict the results of chaotic behavior in a nonlinear system, because no device can store an infinite amount of information. After students have experimented with nonlinear systems, students may see that there exist systems that cannot be perfectly modeled, even with computers.

Sub-hypothesis H2c, "sensitivity to initial conditions": During the intervention, students will learn the concept of "sensitivity to initial conditions," and afterward they will have a greater tendency to believe that small influences in a system can sometimes produce large changes in the future behavior of the system.

Gleick (1987) states that the assumption that "very small influences can be neglected" once lay "at the philosophical heart of science." If traditional physics instruction leads students toward this assumption, can an educational intervention allow students to see that it is not always true?

Sub-hypothesis H2d, "examples of chaos": Before the intervention, students will have difficulties providing examples of systems in which sensitivity to initial conditions occurs. After, students will be able to provide physics definitions of "initial conditions," "chaos," and examples of chaotic systems. Some students will be able to demonstrate their comprehension of these concepts in writing.

Sub-hypothesis H2e, "holistic view": The educational intervention will cause students to move away from reductionist epistemologies and adopt a more holistic view of physical systems.

Holism is the idea that all the properties of a given system cannot be determined or explained by the sum of its component parts alone. Instead, the system as a whole determines how the parts behave. Reductionism is the view that the nature of complex things can always be explained by more fundamental things, such as a system's components. In the philosophy of science, advocates of holism often cite chaotic systems as examples of phenomena that cannot be adequately explained by reducing them to their components. Advocates of reductionism argue that chaotic systems can still be reduced to their parts, though the way those parts interact with each other spawns emergent properties.

Sub-hypothesis H2f, "few variables": Before the intervention, students believe chaotic systems must have many variables. After interacting with physical systems with few variables, students realize that chaotic systems with few variables can exist.

This sub-hypothesis was inspired by the work of Nemirovsky (1993), who found differing views among students during their explorations of a chaotic water wheel.

Sub-hypothesis H2g, "limited to probability": Before the intervention, students may believe that a probability is never all that can be known about a system. The intervention will show students that sometimes in physics, it is only possible to know the probability that something will happen.

Introductory instruction in quantum mechanics commonly states that by the Heisenberg Uncertainty Principle, it is only possible to know the probability that an electron is some distance away from the atomic nucleus.[4] Even if students have heard about uncertainty in the atom, they may not necessarily believe that, epistemologically, a probability is all that can be known about a macroscopic system's future behavior. The intervention may change students' views because, in the magnetic pendulum, the probability of the pendulum bob ending up above a given magnet is 1 divided by the number of magnets (as long as the magnets are an equal distance apart and an equal distance from where the bob would rest without any magnets). However, the path of the bob—and the magnet below the bob when it comes to rest—is not predictable.

Hypothesis H3, "complexity": During the intervention, students will see how nonlinear systems exhibit both order and chaos, with windows of key variable ranges that result in periodicity mapped within variable ranges resulting in chaos.

This pattern is sometimes called "complexity."[5] Order and periodicity are frequently observed in "windows" of variable ranges above and below variable ranges

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[4] Probability in quantum mechanics may be the result of non-deterministic or random processes. Chaos, on the other hand, is not randomness; it is deterministic.
[5] Prigogine (2003) writes, "Complexity is a property of systems that for given boundary conditions have more than one possible solutions. Also in complex systems long range correlations appear between components for very short-range local interactions." A competing academic definition of "complexity" views systems with more variables as

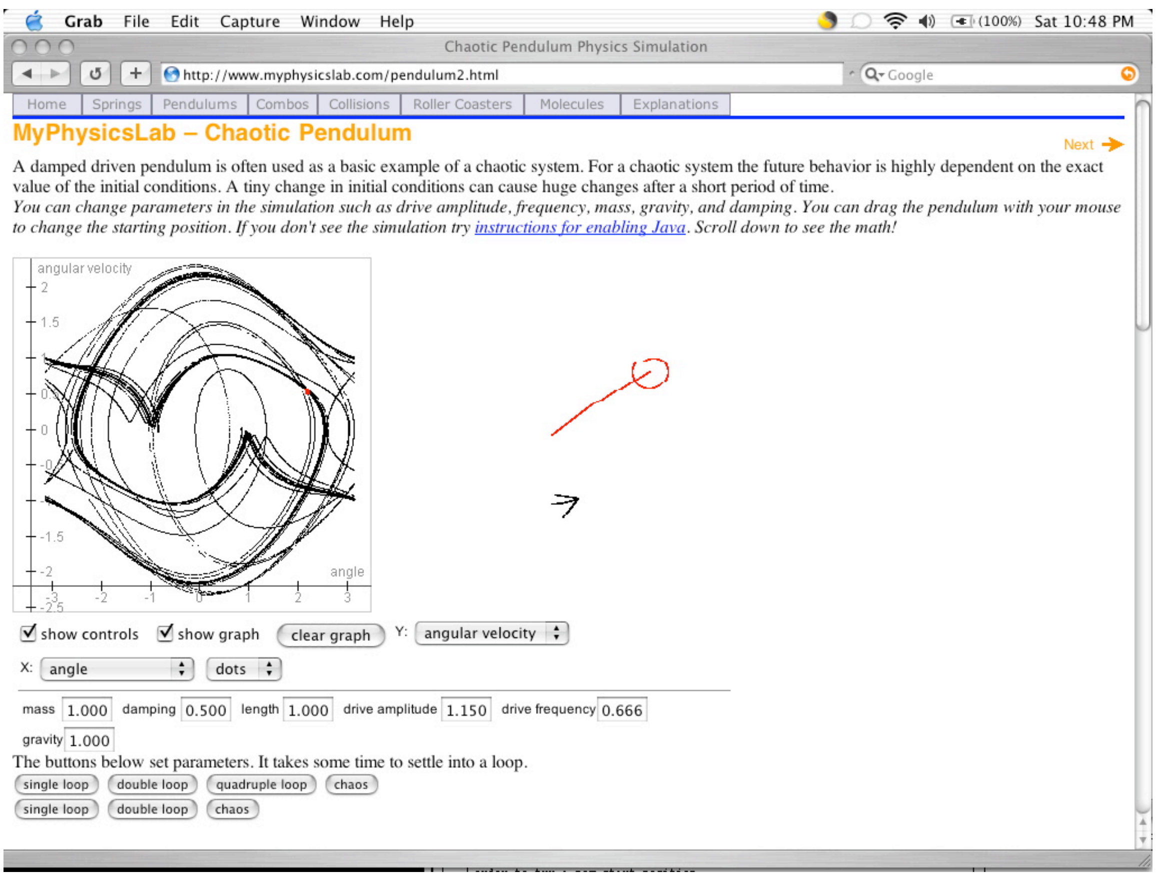
that produce chaotic, unpredictable behavior. This has been known since the 1960s, and was identified in Lorenz's investigations of heat conduction. It can also be seen in graphical form in the Logistic Map.[6]

Hypothesis H4, "deterministic chaos": The computer simulation will allow students to see examples of *deterministic* chaos (Prigogine, 1997). When two starting positions are infinitely identical, trajectories will be the exactly same, even where arbitrarily close starting points produce drastically different results.

Placing the chaotic pattern in multiple embeddings, students will see that determinism arises in the computer simulation although it cannot be detected in students' investigations of the real world pendulum.

Hypothesis H5, "phase space": Students will construct and understand graphical representations in phase space.

Phase space is a graphical representation in which velocity (or angular velocity) is plotted on the y-axis and position (or angle) is plotted on the x-axis. Below is a phase space plot of a chaotic pendulum:



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more complex. Do complex *linear* systems exist in nature? Is a computer a complex linear system?

[6] See http://en.wikipedia.org/wiki/Logistic_map and http://mathworld.wolfram.com/LogisticMap.html for educational resources.

Picture 1. A screen shot from http://www.myphysicslab.com/pendulum2.html shown to students on January 13th, 17th, 18th, and 19th.

Assessments

In the pre-assessments, students were asked the questions, “What is a physical system?” (A1) and “What is a system in physics?” (B1). In the post-assessment, students were asked, “What is a system?” (E7,F6). In hindsight, it was an error to omit the world “physical” in the post-assessment. The purpose of pre-assessment questions A1 and B1 was to identify students’ incoming notions of a physical system. The concept of a system is fundamental to understanding nonlinear systems. Questions A1 and B1 were necessary to determine if students were formulating concepts of a “system” throughout the intervention, or if they came in with clear notions of what a system is in physics.

Pre-assessment items A2, B2, A3, B4, A4 and B3 were designed to test hypothesis H1, students’ epistemological views on predictability prior to the intervention. In A2, students were given the question: “True or False: Some physical systems are, by their nature, unpredictable,” and asked to give the fraction of physical systems that are unpredictable by circling “None,” “A Few,” “Some,” “About Half,” “Most,” “Almost All,” or “All.” In pre-assessment B2, the statement was changed to: “True or False: According to physics, everything is predictable. If you answered “false,” what fraction of physical systems are unpredictable? Circle one.” The survey instruments were designed with the impression that students tend to give affirmative answers to difficult questions. Since on B2, a “false” response was the same as a “true” response to A2, the strength of such an effect could be documented. If an equal percentage of students answered “false” on B2 as answered “true” on A2, one could be reasonably sure that students did not tend towards giving positive responses.

In pre-assessment questions A3, students were asked, “Can you think of a system in which a small change in a variable could produce a completely different future outcome? If possible, provide an example.” In question B4, the question was reworded as, “Can you think of a system in which a very small change in the values of the givens (the “initial conditions”) could result in a completely different answer?” In the A4 and B3, students were given the situation: “Say you solved a physics problem using a set of givens. (For example, v = 1.0000000000 m/s.) Then, you solved the same physics problem using a new set of givens in which the numbers are only slightly different. (For example, v = 1.0000000001 m/s).” Then, students were asked if it is mathematically possible to get a very different answer if initially given values are only slightly different.

Pre-assessment questions A2 and B2 were also designed to test sub-hypothesis H2a (limited prediction) in conjunction with the identical post-assessment items E2 and F2, respectively. A greater number of students viewing some systems as unpredictable would support the sub-hypothesis. Qualitative data from post-assessment questions E5 (“What was the most important thing you learned from the chaos project?”), F5 (“What was the most interesting thing...”), E9 (“What is chaos?”), and F8 (“What is chaos?”) were also obtained to test H2a (limited prediction).

Pre-assessment questions A3 and B4, and A4 and B3, were designed to test sub-hypothesis H2c (sensitivity to initial conditions) in conjunction with post-assessment items E12 and F11. In the post-assessments, students were asked, “What happens when a system's behavior is very sensitive to initial conditions?" (F11) and to provide an example of "sensitivity to initial conditions" (E12). Sub-hypothesis H2c (sensitivity to initial conditions) would be supported by pre-assessment responses indicating that small changes cannot produce different outcomes together with post-assessment comments stating the opposite. Post-assessments E8/F7 ("What is an initial condition?") were also designed to provide data that might support H2c.

In a follow-up question, students were asked to give examples of chaotic systems that were not studied during the project and to explain why they are chaotic (E13, F12). These follow-up questions were designed to test sub-hypothesis H2d (examples of chaos) in conjunction with pre-assessment questions A3 and B4. A lack of examples of scientifically chaotic systems in the pre-assessments combined with student examples of chaotic systems in the post-assessment would support the sub-hypothesis. Post-assessment items E9/F8 ("What is chaos?") and E12 were also designed to provide data that might support H2d (examples of chaos), or to aid in the interpretation of other results.

Five Likert questions were identical in the pre- and post-assessments. A 9-point scale was used, in which 1 represented "Strongly Agree" and 9, "Strongly Disagree."

Likert questions A5/E14 ("If enough information is known about a system, it is possible to predict everything that will happen in it") and D6/F15 ("Nature has "built-in" limits to what it is possible to know about some physical systems") were designed to test sub-hypothesis H2a (limited prediction). More disagreement in E14 compared to A5 and more agreement in F15 compared to D6 would support this sub-hypothesis.

Likert questions C8/E17 ("If you analyzed a coin toss with scientific instruments, you could predict the outcome every time"), D8/F17 ("It is always possible to know something with absolute certainty, if one has the right tools or measuring devices"), and C6/E15 ("Anything in nature can be accurately modeled with computers") were designed to test sub-hypothesis H2b (modeling uncertainty). A greater degree of disagreement with these statements in the post-assessment would support this sub-hypothesis.

Likert items C7/E16 ("Small influences in a system, such as air currents in a room, cannot produce large changes in the future behavior of the system") and D7/F16 ("Small influences in a system, such as air currents in a room, can produce large changes in the future behavior of the system") were designed to test sub-hypothesis H2c (sensitivity to initial conditions). More disagreement with E16 compared to C7 and more agreement with F16 compared to D7 would support this sub-hypothesis.

Likert items C9/E18 ("In some systems, the system's behavior cannot be modeled by studying the system's parts. The system must be studied as a whole") and B5/F14 ("In physics, the universe is analyzed by breaking it down into its component parts, just as one can figure out how a machine works by finding the purpose of each of its parts") were designed to test sub-hypothesis H2e (holistic view). More agreement with E18 compared to C9, and more disagreement with F14 compared to B5 would support this sub-hypothesis.

Likert items D9/F18 ("In some systems, it is only possible to know the probability that something will happen") were designed to test sub-hypothesis H2g (limited to probability) along with data from true or false questions E4 and F4. Question E4 asked, "Sometimes, physics can only predict the probability that something will happen, no matter how accurately things are measured." Question F4 stated, "Probability is the result of the laws of nature." Affirmative answers would support the hypothesis.

Post-assessment true or false questions E1 and F1 were designed to test sub-hypothesis H2f (few variables). Post-assessment question E1 stated, "For a system to be chaotic, it must have many variables," and F1 read, "Can a system with only a few variables exhibit chaos?" Answers of "false" for E1 and "true" for F1 would support the sub-hypothesis, although comparison data was not obtained in pre-assessments.

True or false questions E3 and F3 were designed to test hypothesis H3 (complexity) by asking students if a chaotic system can be ordered or periodic. Affirmative answers would support the hypothesis. A better phrasing would have asked if a nonlinear system can be ordered or periodic, but the term "nonlinear" was not introduced to students.

Qualitative data (students interviews, oral and written comments) were used to test hypotheses H4 (deterministic chaos) and H5 (phase space) and to supplement quantitative data for all other hypotheses and sub-hypotheses.

Curriculum Description

January 11, 2006

On Wednesday, January 11, 2006, pre-assessment surveys were collected and discussed in class. Students brought up weather and chemical reactions as examples of an unpredictable systems. The instructor posed the question, "Will we ever be able to accurately predict the weather?" Some students felt that some systems, like weather, are inherently impossible to predict. For example, BS said that since there are so many constantly changing variables, there is no way we will ever be able to accurately predict it. Other students thought that increasingly accurate predictions will be possible as technology improves. When the instructor asked if a small change, like moving one's hands, could cause a tornado, MSt brought up the example of a bomb in which "a small thing triggers a big explosion in a chain reaction." Seven students voted that they doubted if a small change could produce a large effect. One said that it depends on the situation. The instructor acknowledged that it depends on what kind of system is being studied. QP said that something relatively minor, like the temperature, might influence the outcome in a race.

The dueling calculators activity sheet was distributed. The instructor asked for a random number; a student volunteered 510. Half of the class—those with odd birthdays—was instructed to enter 0.510 into their calculators. The other half was told to enter 0.50999999999, and to keep typing 9s until they could not enter any more digits. They wrote this number on their worksheets as the "initial number." In degree mode, students multiplied this number by 180 and hit the SIN key to take the sine of the product. Students were asked to make a hypothesis about what will happen when these last two steps are repeated 25 times. Students did so and recorded their results.

Students' hypotheses varied. Six students indicated that the numbers will decrease, three that they would increase, and two said that they would change. Three said that the 25th iteration would be the same as the initial number. Eleven students gave hypotheses that compared the two initial numbers. Of these, three wrote that both sets of results would be the same, three that they would be slightly different, and four indicated that there would be two different sets of numbers. One student, MSt, predicted, "The numbers will differ more and more every iteration, the 25th iterations will be different." MSt's comment is noteworthy because he was able to provide a somewhat accurate prediction of the results of the activity.

Students were told that all results should be in between 0 and 1. When finished, students compared their results. One group found that their results were similar until the eighth iteration; then they began to diverge rapidly. AB said that this was because of rounding. Through questioning, students concluded that the calculator has finite (limited) memory. CP said that the calculator has to round the numbers; the instructor explained that this is true—since each number is infinitely long, the calculator must round. MR and QP pointed out that all of their numbers were the same. The instructor pointed out that they used the same calculator, and asked, "What's different between two calculators?" LR said that one has more memory, and the instructor acknowledged that the number of decimal places to which the number is rounded varies from calculator to calculator. The instructor pointed out that two students with the same brand of calculator got the same set of answers. Students were asked to write down the make and model of their calculators.

Students did not appear to be very surprised by these findings, even thought they had been rounding their answers in traditional physics problems throughout the semester. The concept of "sensitivity to initial conditions" was introduced. Students then answered the question, "Did your neighbors get similar results? Record any similarities or differences." Five groups of students found that results were the same until the 3rd, 8th, 13th, 15th, and 23rd iterations, respectively. Students with the same make and model of calculator found that their results were the same.

Students also wrote their conclusions. Out of 28 students, twelve (43%) indicated that the results depended on the kind of calculator used, in keeping with sub-hypothesis H2b (modeling uncertainty). Ten students indicated that the calculators' rounding processes caused an effect, and seven explicitly stated that a slight difference in the initial number would change the final outcome; thus, 61% of students provided evidence for H2c (sensitivity to initial conditions).

When asked, "What did you learn from this activity?", twelve out of 28 students mentioned that different calculators round differently, and three said that calculators are imperfect. Thus, 54% of students provided evidence supporting H2b (modeling uncertainty). Seven (25%) stated that small initial differences can result in big differences in the end, providing evidence for H2c (sensitivity to initial conditions). It is not surprising that more students mentioned differences in calculator rounding algorithms than mentioned initial differences in seed values, because even two identical seed values could produce divergent outcomes in two different calculators.

The final question on the worksheet was, "Write down another example of 'sensitivity to initial conditions.'" Two students mentioned a "weather forecast," and another stated that if one person on a sports team has a bad day, the whole team can have a bad game. Interestingly, one wrote, "Where your TV antenna is at. My sister has better reception than me and our rooms are not that far." Another stated that two different processes can produce the same result using an example of hitting a tennis ball. Five students suggested using another mathematical function such as cosine instead of sine.

Taken together, these responses provide qualitative evidence for sub-hypotheses H2a (limited prediction), H2b (modeling uncertainty), H2c (sensitivity to initial conditions), and hypothesis H4 (deterministic chaos). In keeping with H2a, students were able to see that the outcome of the algorithm was unpredictable. In keeping with H2b, students realized that any given calculator was imperfect. The comments of many students explicitly mentioned the concept of sensitivity to initial conditions, or alluded to it through stating that a rounding can effect results, in keeping with H2c. Qualitative evidence of students' seeing that the same make and model of calculator gives the same results provides tentative support for H4.

January 12, 2006

On Thursday, January 12, 2006, each of the four groups of four students was divided in two. Two students from each group completed the bowl activity, and engaged in the faucet activity. Students conducted inquiry based investigations and were encouraged to explore. These activities were only marginally successful because students had difficulties seeing their connections with the other activities in the intervention.

At one point during the bowl activity, the instructor asked, "What would be a small external force on the marble?" Students could not answer the question, an unsurprising result given that people remain consciously unaware of the many small forces about us all the time. (A common example of this given in physics instruction is the Earth's magnetic force field.) While pointing at an air vent, the instructor offered, "the wind currents in the room?" Students understood that air currents could exert small forces, because this topic had been covered in lessons about air resistance and drag forces.

During the faucet activity, LR said that as the handle is turned, "the more pressure, it fills up the hole, [pointing to the faucet]- I'm not sure how to say it." She wrote, "when you gradually turn the knob more, the shape of the water is more defined." The activity instructions were unclear as to whether the pattern involved the initial creation of the water flow in addition to the way in which the flow changes as the handle is turned. Students were meant to focus on the latter effect, but often did not. The question of "cut-off time" has been an issue in the Patterns research group on several occasions. When identifying a pattern, it is reasonable to ask, "when does the pattern begin, and where does it end?"

January 13, 2006

On Friday, January 13, 2006, Instructional materials were passed out to groups 1,2,3 and 4 (16 students) with pictures of fractals from Burger and Starbird (2000). The instructor set up ring stands with materials for a simple pendulum. Students began by analyzing the mechanical energy dynamics in the simple pendulum, a tie-in with a topic that had just been covered in the class curriculum. Next, students were given the problem drawing the path of the pendulum bob in phase space, provided with graph paper and labeled axes. At first, instructor scaffolding was required. When the instructor explained the negative and positive directions and origin (0,0) on the position and velocity axes, Group 1 (QP, MR, AP, and JM) grasped the concept of phase space:

MR: It’s a circle.
AP: it’s going to be a shape/ a circle?
Primary Investigator: [nods] yes, but it eventually stops, so what’s happening
AP: It gets smaller.
MR: So it’s really a spiral.
PI: yes!
At this point, MR expresses confusion. His prior idea that the y-axis represents y-displacement shows robustness:
MR: All this is x-axis? There is no y-axis?
QP explained why it is a spiral. By this point, the group of four students could see as it goes back and forth, it goes from + to - in position and velocity.

This is qualitative evidence that changing the axis labels confuses students. For some reason, students have difficulties understanding graphical representations when axes are defined in new or unfamiliar ways. It was observed in almost all other student groups, with some notable exceptions. It is a strong argument for further research in teaching students literacy skills in graphical representations. Even in a UC Berkeley research group, participants could not see the spiral without some hints, so it is not surprising that the high school students needed scaffolding. But significantly, since the answer requires no advanced knowledge in physics, students eventually identified the circle and spiral attractors: “it’s going to be a shape/ a circle?”

Next, JM said: “It starts from negative. It’s going around the wrong way.” Now, students debated whether the direction of the spiral should be clockwise, or counterclockwise, a non-trivial question.[7]

Next, students were given magnets to place under their pendulums. Inquiry-based student investigation was facilitated, with students writing hypotheses prior to experimentation.[8][9] Students placed one magnet under their pendulums, creating nonlinear systems. Before releasing the pendulum’s bob, all students’ hypotheses predicted linear behavior in keeping with hypothesis H1 (prior predictability). Significantly, no responses indicated anything about possible chaotic or random behavior. “Unpredictability” was not mentioned by any students.

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[7] The answer depends on whether the bob’s initial velocity (after its initial angular displacement, a debatable part of the pattern) is to the right (positive) or to the left (negative). If the right is negative and left is positive, it is the opposite sign.

[8] When facilitating this lab in the classroom, physics instructors should note that chaotic behavior is the most obvious when the distance between the bob and the magnets is the smallest. Magnets should be firmly attached to the table or ring stand base with tape or, if necessary, glue, so that they do not jump up and stick to the bob. Round circular magnets work best, and the north and south sides should be labeled with paint or correction fluid. Attaching the magnets to the base of a ring stand changes the physics of the system, since the ring stand can resonate. Two or more magnets are necessary to see obviously chaotic effects.

[9] See http://www.exploratorium.edu/snacks/strange_attractor.html for an educational resource.

In their hypotheses, nine students predicted the pendulum bob would slow down, but five wrote that it would speed up, and one that it would "not slow down." Four said that the magnet would attract the bob; in contrast, four predicted that the bob would not stop moving. Three students indicated that the "pattern will change," and another two that the bob would stop in the middle. Other individual responses included "no effect," "the shape created by the bolt becomes flatter," and "the magnet will stick to the bob." Each of the n=33 responses was coded into one of these ten categories. In future work, students could be given a more specific question, such as one asking if the bob's motion will become less predictable.

Before students placed a second magnet below the pendulum, they were asked to give another hypothesis about what would happen.[10] Again, no students indicated that "chaotic," "unpredictable," or "random"[11] behavior would occur in the system, evidence for hypothesis H1 (prior predictability). Only one student's (MSt's) answer gave any indication of there being limits to what is possible to know: "stops at one of the magnets, don't know which." One other student (JCh) hinted at strangeness, writing that the magnets give the bob "two jerks" when it passes over them. DS acknowledged, "If there are two magnets, then the magnets will attract the bob with the same force." Ten students stated the modal response, that the bob will slow down.[12] The remaining 20 students gave a wide range of answers, such as "stick together" (n=3), "speed up" (n=2), move "back and forth" (n=2), "swing between both" (n=2), "stay at constant speed" (n=2), "pattern will stay but the distance will decrease" (n=1), "orbit" (n=1), and "elipse" (n=1). Significantly, 30 out of 33 student responses (91%) were descriptions of predictable, linear behavior—strong evidence for hypothesis H1 (prior predictability).

For example, in Group 1, AP wrote: "It will continue in orderly fashion back and forth from magnet to magnet." Her group then observed the motion of their chaotic magnetic pendulum (0:50):

JM: It's out of control / Chaos / I think it's going to stay like that for a long time.
MR: It will never stop / like metal balls.
PI: Is it repeating the same-?
Ss: No.
JM: maybe it's random.
PI: Could you predict what it's going to do?
Ss: No.
PI: but now it's going back and forth / so it is periodic?
[Then, the pendulum stopped acting periodically. Everyone sees this.]
PI: now, it's chaotic

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[10] When two magnets are placed below the pendulum, the system's dynamics are such that a tiny change in the placement of a magnet can radically change the bob's trajectory, as can two nearly identical releases from arbitrarily close starting positions.

[11] Random behavior is not chaotic, although chaotic behavior may appear to be random because it is unpredictable. Students who acknowledged randomness were correct in noting the unpredictable nature of the system, so their responses were interpreted as supporting H2a (limited prediction).

[12] This is not an incorrect answer, because energy dissipates in a non-linear pendulum just as it does in a simple pendulum.

It is a property of nonlinear, chaotic systems that, under certain ranges of the essential variables, they can exhibit ordered, periodic behavior. But the students who observed the magnetic pendulum behaving periodically did not write answers supporting hypothesis H3 (complexity), which states that students were able to understand that non-linear systems can exhibit order. In future work, further scaffolding could facilitate understandings of this concept. However, JM did write "It's random," in keeping with H2a (limited prediction).

JM's comment "I think it's going to stay like that for a long time" is interesting. The instructor was not able to get JM to defend this initial hypothesis, because it was quickly disproved by direct observation. However, JM's assertion may be an acknowledgement of the way that certain non-linear systems can sustain ordered, periodic behavior, far from equilibrium, for long periods of time. Such "dissipative structures" were discovered by Ilya Prigogine, and examples range from weather and chemical systems[13] to living organisms themselves. [14]

In Group 2 (KC,DS,PA, and FR), students realized that the simple pendulum is not sensitive to initial conditions, noting that "there is no difference" when the bob is released from similar starting positions. After placing down two magnets, students had varying hypotheses. DS's hypothesis, that the two magnets "will attract the bob with the same force," acknowledged the unstable equilibrium in the system. PA wrote, "I think the bob will move around." Upon observing the bob's chaotic trajectory, DS wrote: "The bob revolves & swings uncontrollably because the bob is looking for which magnet has a stronger attraction." KC wrote: "it moved out of control when the bob and magnite [sic] were close together but it's still swinging." FR wrote something similar: "The pendulum attracts to either one of the sides of the magnet and moves out of control."[15] The responses of DS, KC, and FR are in keeping with H2a (limited prediction).

After observing chaos, students in Group 2 performed the experiment of releasing the bob multiple times from the same starting point. After one release, they recorded their hypotheses regarding the next release. DS did not seem to understand that the question was asking about how the bob's swing would be different in consecutive trials: "it will move in two directions, back & forth." PA wrote, "I think the bob will follow the same path." KC wrote that as well, but added, "As it gets closure [sic] to the magnitude [sic] it will go chaotic." FR wrote, "The same pattern will keep on repeating because of the magnet." [16]

In his observations, PA acknowledged that his hypothesis was disproved: "The bob did not follow the same path and continued to move," qualitative evidence for sub-hypothesis H2c (sensitivity to initial conditions). KC wrote, "As the ball got closer to the

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[13] See http://people.musc.edu/~alievr/BZ/BZexplain.html for an educational resource.

[14] See http://www.prototista.org/E-Zine/OriginsofOrder/OriginsofOrder-TOC.htm for an educational resource.

[15] The use of the phrase "out of control" to describe chaos is conceptually interesting. Is chaos an uncontrollable pattern? In some ways it is, although contemporary research in chaos theory does focus on ways to control chaotic systems by modeling them mathematically.

[16] Future work could involve further thinking about exactly what students meant when they wrote these statements.

magnitude [sic] it did go chaotic." Significantly, KC was comfortable enough with the physics definition of "chaotic" to use it to accurately describe a chaotic system—evidence for sub-hypothesis H2d (examples of chaos). FR recorded: "The same pattern did occur again b/c of the magnet's attraction." Interestingly, FR chose to conceptualize chaos as a "pattern," which is also how the UC Berkeley Patterns Research Group sees it. He also noticed that the magnets played a role in the pattern's existence. In a follow-up experiment, FR wrote that the overall "pattern" of the bob's motion did not change when a small wind force was exerted on it. He did not pay attention to the intricacies of the bob's trajectory, and therefore wrote that a "small external force does not affect it significantly." Although his conclusion was opposite to the one students were supposed to come to, from the perspective of the Patterns Research Group, it is not surprising given that FR viewed chaotic motion as a "pattern." Certainly, chaotic motion has a recognizably different structure than harmonic oscillation or linear motion. FR's comment indicates that he was aware of some of these differences.

January 17, 2006

Dr. Martin Luther King, Jr.'s birthday was celebrated on Monday, January 16, 2006. On Tuesday, Groups 5,6,7 and 8 were given handouts with instructions similar to those given on January 13, but with minor corrections and revisions. First, students built simple pendulums and drew the bob's trajectory in phase space. One group of students (Group 5: CC, TL, IC, and SS) were able to draw a circle in phase space with no assistance from the instructor:

PI: Why did you say it was a circle?
TL: symmetry
PI: Over the long term, what happens?
CC: It's a spiral

As expected, these students observed no differences in the trajectory of the simple pendulum when it was released from two similar positions. However, their responses indicate that they did not understand the meaning of the phrase "sensitivity to initial conditions" at this point in the intervention.

Later, these students wrote hypotheses predicting what will happen when two magnets are placed under the bob. All four indicated that the "magnets will slow the bob down." These students did not predict chaos or random behavior, evidence supporting H1. From the perspective of the Patterns Research Group, these students gave a mechanical rather than a "patterns level" type explanation.

When the group's magnetic pendulum began acting chaotically, IC appeared to be amused by its unpredictable trajectory, evidence that its behavior acted as a discrepant event to facilitate conceptual change. Her words and emotions indicated her surprise that a simple nonlinear system can behave strangely: "Look, it twists / oh its going to this way, see, it's turning around, now, look at it! I'm so proud of it...now it stopped." [17] For her observations, she was unfortunately epigrammatic, writing, "The magnets slightly push the bob."

[17] IC's comment may be an example of "magical thinking." To her, the pendulum seems to take on a life of its own.

This group devised their own experiment, placing two magnets one inch apart and releasing the bob from a height of 5.5 cm directly above a point on the table. Prior to release, two students hypothesized that the bob would end up in the middle, between the two magnets. IC wrote, "I think that small forces can significantly change the behavior," and SS indicated that the bob will stop at a "random" magnet depending on "when it runs out of energy." The latter two responses support sub-hypothesis H2c (sensitivity to initial conditions). The students observed that the bob "randomly chose one magnet to stop at," and when the experiment was repeated, "it chose the other magnet." CC and SS called this phenomenon "random selection," TL wrote "arbitrary selection," and IC stated that "there isn't a 'constant' answer, it's unpredictable and random." These experimental conclusions clearly indicate that these students underwent conceptual change as predicted by sub-hypotheses H2a (limited prediction) and H2g (limited to probability).

Another group of students (Group 6: CP, JCh, MT and IP) hypothesized that after placing one magnet below the pendulum bob, "The bob's swing will be interrupted by the force of the magnet and make the swing different." They noticed that when the bob is close to the magnet, it "gets a little jerk and the swing is bumpy." They predicted that two magnets will produce two jerks. Then, these students observed chaos:

PI: Do you think you could predict which one it ends up at?

JCh: No.

PI: This is chaos, because it's very sensitive to initial conditions. We cannot predict which magnet it will end up at.

MT: that's cool!

This group of students, bored and frustrated until this point, immediately became amused and excited by physics. Interestingly, students with dueling hypotheses began to bet each other about which magnet the bob would end up over. As energy dissipated, the bob circled around one magnet, then moved to the next, with a student advocating for it to stop at the magnet he bet on. Students clearly realized that they each had 50/50 odds; a probability was all they could know. All four students in this group wrote that the bob's movement was "unpredictable." One also wrote that it was "chaotic." This is qualitative evidence for sub-hypotheses H2a (limited prediction), H2d (examples of chaos), and H2g (limited to probability).

Immediately afterward, these students predicted what would happen if the bob were to be released from approximately the same place twice. Interestingly, students wrote, "it will follow the same path as before," indicating robustness in the view that small changes cannot have divergent effects. After doing the experiment, students saw that their hypothesis was disproved, as evidenced by the group's dialog:

PI: Even if you released from the same point in space, is there any way of knowing where it's going to land?

JCh, MT, IP: [together] no!

Thus, students were led toward sub-hypothesis H2c (sensitivity to initial conditions).

In Group 7 (JG, CS, BS and MS), CS predicted that the bob would stop at the same magnet it stopped at the last time it was released. When it did, BS noted: "But it didn't follow the same path." Then, hypothesizing what will happen if the bob is released from approximately the same place, BS wrote, "It will go to either of the magnets, can't tell which," in keeping with H2a (limited prediction) and H2c (sensitivity to initial conditions). His partner MS hypothesized that "it will do the same thing," but observed that the bob followed a different path, concluding, "initial determines final," in keeping with H2c. On videotape, MS and BS appear surprised and confused by the pendulum's unpredictability. At the same time, they can see that unpredictability results from the trajectory's sensitivity to the precise initial conditions of its two approximately identical starting locations.

In Group 8, students (AB, JQ, ED and JC) observed no sensitivity to initial conditions in the simple pendulum, as expected. This group predicted that placing magnets under the pendulum would make it "slow down." Students in this group were divided; one, JQ, had difficulties accepting sensitivity to initial conditions and unpredictability: [18]

PI: If you were to do that experiment over again...do you think it would end at magnet B again?
AB: no
JQ: yes
PI: So what's going to determine if it ends up at A or B?
JQ: Where you release it from
PI: Do you think it would be easy to determine where it ends based on what point you release it from?
JQ: yes
AB: no
PI: Do you think if you release it from a point around this one [pointing at one of the magnets] it will end up at that one? [releases the pendulum]
JQ: yes
AB: no, because this one will slow it down, and it will go to this one [pointing at the other magnet. As if on que, the bob does this.]

The above dialog shows that students had differing views on predictability, offering inconclusive evidence for H1 (prior predictability). JQ robustly held on to the view that the bob would follow the same path as before, but AB was more willing to accept unpredictability as natural. The extent to which students like AB's epistemological views were altered is unclear, but JQ's surely were, as his observations disproved his predictions. Since his observations contradicted his epistemological beliefs, conceptual change occurred, evidence for H2a (limited prediction).

Later, this group of students released their bob from approximately the same set of initial conditions (0:52):

PI: Did it follow the same path?
AB: kind of
PI: kind of?
AB: no
IP: try it again
PI: Is this motion it's doing now the same as it was before?
AB: It looks kind of the same, but not really
JQ: it didn't do that (before), [pointing to the bob going around in circles]
ED: I guess because it's not launched from high

[18] See videotape 1/17B starting at 0:36:00 for further reference.