AMPERE's LAW

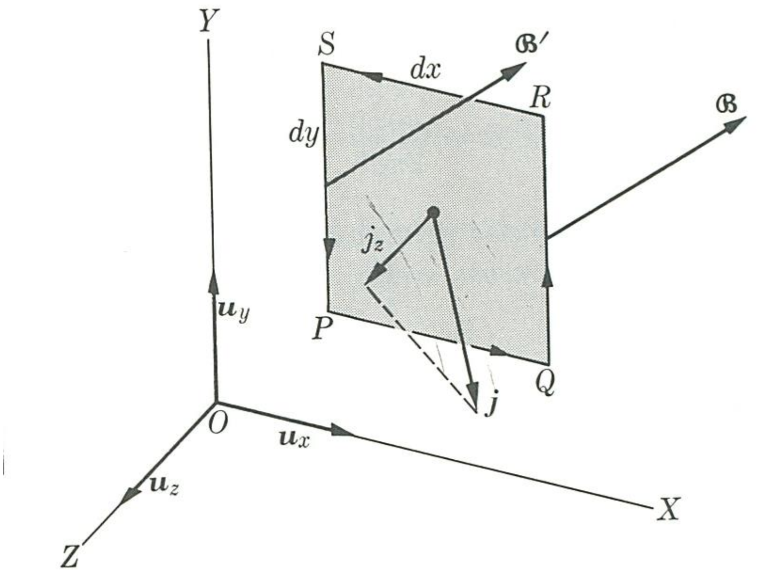

Fig. 16–42. Elementary path to evaluate Ampère's law in differential form.

Since we know that Ampère's law can be applied to a path of any shape, let us apply it to a very small or infinitesimal rectangular path $PQRS$ in the XY-plane, having sides dx and dy and an area $dx\,dy$ (Fig. 16–42). The sense of circulation around $PQRS$ is as indicated by the arrows. The circulation $\Lambda_{\mathcal{B}}$ consists of four terms, one for each side; that is,

$$\Lambda_{\mathcal{B}} = \oint_{PQRS} \boldsymbol{\mathcal{B}} \cdot d\boldsymbol{l} = \int_{PQ} + \int_{QR} + \int_{RS} + \int_{SP}. \tag{16.72}$$

Now along the path QR, which is oriented parallel to the $+Y$-direction, $d\boldsymbol{l} = \boldsymbol{u}_y\,dy$ and

$$\int_{QR} \boldsymbol{\mathcal{B}} \cdot d\boldsymbol{l} = \boldsymbol{\mathcal{B}} \cdot \boldsymbol{u}_y\,dy = \mathcal{B}_y\,dy.$$

Similarly, for side SP, which is oriented in the $-Y$-direction, $d\boldsymbol{l} = -\boldsymbol{u}_y\,dy$, and thus

$$\int_{SP} \boldsymbol{\mathcal{B}} \cdot d\boldsymbol{l} = -\boldsymbol{\mathcal{B}}' \cdot \boldsymbol{u}_y\,dy = -\mathcal{B}'_y\,dy,$$

so that

$$\int_{QR} + \int_{SP} = (\mathcal{B}_y - \mathcal{B}'_y)\,dy.$$

But, since $PQ = dx$, $\mathcal{B}_y - \mathcal{B}'_y = d\mathcal{B}_y = (\partial\mathcal{B}_y/\partial x)\,dx$. Therefore

$$\int_{QR} + \int_{SP} = \frac{\partial\mathcal{B}_y}{\partial x}\,dx\,dy.$$