Section 52. Power Series

Do problems 3-33 (odds) and 41 from Stewart, Section 12.8 (Power Series)

Section 53. Representing Functions with Power Series

Do problems 3-17 (odds), 23-33 odds and 37 from Stewart, Section 12.9 (Representations of Functions As Power Series)

1. Differentiate the Geometric Series:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \ldots$$

to get a function that represents the series:

$$\sum_{n=1}^{\infty} nx^n$$

Find its interval of convergence. Why can we start this series at 1 instead of 0?

2. Use the previous Exercise to find the exact sum of the series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{5^n}$$

3. Differentiate the series for $\sum_{n=1}^{\infty} nx^n$ in order to get a function that represents the series:

$$\sum_{n=1}^{\infty} n^2 x^n$$

Find its interval of convergence.

4. Use the previous Exercise to find the exact sum of the series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{5^n}$$

5. Find the exact sum of the series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n (3n^2 - 5n + 2)}{7^n}$$

6. Perform a partial fraction expansion of

$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

in order to find a function that represents

$$\sum_{n=1}^{\infty} \frac{x^n}{n(n+1)}$$

Find its interval of convergence.

7. Integrate the Logarithmic Series:

$$-\ln(1-x) = \sum_{n=1}^{\infty} \frac{x^n}{n}$$

to get a function that represents:

$$\sum_{n=1}^{\infty} \frac{x^n}{n(n+1)}.$$

Hint: you can try Integration by Parts. Is this process easier or harder than the strategy of the previous Exercise?

8. Find the exact sum of the series:

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1) \cdot 3^n}.$$

9. Find a function that represents the series:

$$\sum_{n=0}^{\infty} \frac{3n+1}{n+1} x^n.$$

Find its interval of convergence. Hint: start with a long division.

10. Use the previous Exercise to find the exact sum of the series:

$$\sum_{n=0}^{\infty} (-1)^n \frac{3n+1}{(n+1)4^n}.$$

Answers:

1. $\frac{x}{(1-x)^2}$, $x \in (-1, 1)$.

2. $-\frac{5}{36}$

3. $\frac{1+x}{(1-x)^3}$, $x \in (-1, 1)$.

4. $\frac{25}{54}$

5. $\frac{13}{256}$

6. $1 + \frac{1-x}{x} \ln(|1-x|)$, $x \in [-1, 1)$

7. same answer as 6.

8. $2 \ln \frac{2}{3} + 1$

9. $\frac{3}{1-x} + \frac{2}{x} \ln(|1-x|), x \in (-1, 1)$.

10. $\frac{12}{5} - 8 \ln \frac{5}{4}$