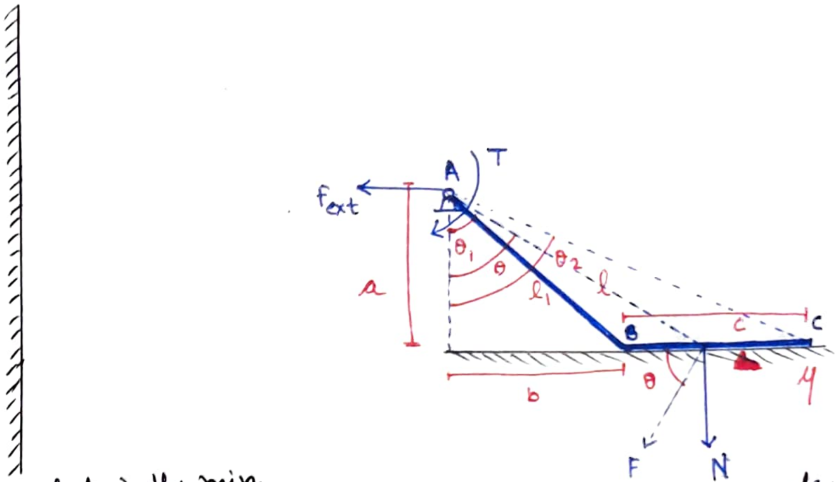


What is the min torque 'T' needed to stop the structure / rod ABC if static coefficient of friction is μ. Assume 'a', 'θ_1', 'θ_2' are given.

→ Rod ABC has a pivot at point A.

→ A force F_{ext} is applied to point A towards left.

Sol^n]

- Calculating Normal force at some random $\theta \in (\theta_1, \theta_2)$

$$l = \frac{a}{\cos\theta}$$

$$b = a \tan\theta_1$$

$$N = F.\sin\theta$$

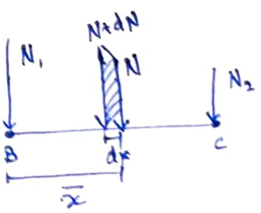
$$= \frac{T}{l}\sin\theta$$

$$\boxed{N = \frac{T}{a}\sin\theta\cos\theta}$$

∴ Normal force at $\theta = \theta_1 \Rightarrow N_1 = \frac{T}{a}\sin\theta_1\cos\theta_1$

∴ Normal force at $\theta = \theta_2 \Rightarrow N_2 = \frac{T}{a}\sin\theta_2\cos\theta_2$

⇒ Consider only segment BC of the Rod:



$$\therefore \bar{x} = \frac{\int x\, dA}{\int dA}$$

& $x = l\sin\theta - b$

$x = a\tan\theta - b$

$x = a\tan\theta - a\tan\theta_1$

$$\boxed{dx = a\sec^2\theta\, d\theta}$$

$$\therefore \bar{x} = \frac{\int x\,(N dx)}{\int N dx}$$

$$\therefore \bar{x} = \frac{\int_{\theta_1}^{\theta_2}(a\tan\theta - b)\left(\frac{T}{a}\sin\theta\cos\theta\right)(a\sec^2\theta\, d\theta)}{\int_{\theta_1}^{\theta_2}\left(\frac{T}{a}\sin\theta\cos\theta\right)(a\sec^2\theta\, d\theta)}$$

$$\therefore \bar{x} = \frac{\int aT\tan\theta\sin\theta\cos\theta\sec^2\theta\, d\theta - \int_{\theta_1}^{\theta_2} bT\sin\theta\cos\theta\sec^2\theta\, d\theta}{\int_{\theta_1}^{\theta_2} T\sin\theta\cos\theta\sec^2\theta\, d\theta}$$

$$\therefore \bar{x} = \frac{\int_{\theta_1}^{\theta_2} aT\tan^2\theta\, d\theta - \int^{\theta_2} bT\tan\theta\, d\theta}{\int_{\theta_1}^{\theta_2} T\tan\theta\, d\theta}$$

$$\therefore \quad \bar{x} = \left. \frac{a(\tan\theta - \theta) - b \ln|\sec\theta|}{\ln|\sec\theta|} \right|_{\theta_1}^{\theta_2}$$

$$= \left. \frac{a(\tan\theta - \theta)}{\ln|\sec\theta|} - b \right|_{\theta_1}^{\theta_2}$$

$$\boxed{\bar{x} = \frac{a(\tan\theta_2 - \theta_2)}{\ln(\sec\theta_2)} - \frac{a(\tan\theta_1 - \theta_1)}{\ln(\sec\theta_1)}} \quad \left[0 < \begin{matrix}\theta_1, \\ \theta_2\end{matrix} < \frac{\pi}{2}\right]$$

$$\therefore \quad \boxed{\bar{\theta} = \tan^{-1}\left(\frac{\bar{x} + a\tan\theta_1}{a}\right)}$$

$$\therefore \quad \bar{N} = \frac{T}{a} \sin\bar{\theta} \cos\bar{\theta}$$

To prevent the rod from moving:

$$\bar{N} y = F_{ext} + F\cos\bar{\theta}$$

$$\therefore \left(\frac{T \sin\bar{\theta}\cos\bar{\theta}}{a}\right) y = F_{ext} + \frac{T}{a}\cos^2\bar{\theta}$$

$$\therefore \quad \cancel{T_{min} = \frac{F_{ext}\, a}{\sin\bar{\theta}\cos\bar{\theta}\, y}} \qquad \boxed{T_{min} = \frac{F_{ext}\, a}{(\sin\bar{\theta}\cos\bar{\theta} - \cos^2\bar{\theta})}}$$