

$$\phi(z) = Az(z-L), \quad 0 < z < L$$

$$\int_0^L \phi^2(z) dz = 1$$

$$\Rightarrow A^2 \int_0^L z^2 (z^2 - 2zL + L^2) dz = 1$$

$$\Rightarrow \int_0^L (z^4 - 2z^3L + z^2L^2) dz = \frac{1}{A^2}$$

$$\Rightarrow \int_0^L z^4 dz - 2L \int_0^L z^3 dz + L^2 \int_0^L z^2 dz$$

$$\Rightarrow \left[\frac{z^5}{5} \right]_0^L - 2L \left[\frac{z^4}{4} \right]_0^L + L^2 \left[\frac{z^3}{3} \right]_0^L = \frac{1}{A^2}$$

$$\Rightarrow \left(\frac{L^5}{5} - \frac{2L^5}{4} \right) + \frac{L^5}{3} = \frac{1}{A^2}$$

$$\Rightarrow \frac{4L^5 - 10L^5}{20} + \frac{L^5}{3} = \frac{1}{A^2}$$

$$\Rightarrow -\frac{5L^5}{20} + \frac{L^5}{3} = \frac{1}{A^2}$$

$$\Rightarrow \frac{-L^5}{4} + \frac{L^5}{3} = \frac{1}{A^2}$$

$$A = \left(\frac{-L^5}{4} + \frac{L^5}{3} \right)^{-1/2}$$