We assume two galaxy population, A and B; the corresponding maps have the following $a_{\ell m}$:

$$a^{A}_{\ell m} = b_A\, a^{M}_{\ell m} + a^{pA}_{\ell m} \tag{1}$$

$$a^{B}_{\ell m} = b_B\, a^{M}_{\ell m} + a^{pB}_{\ell m} \tag{2}$$

Here, b_A and b_B correspond to the linear galaxy bias for both populations, $a^{pA}_{\ell m}$ and $a^{pB}_{\ell m}$ are the respective Poisson contribution to the maps, and $a^{M}_{\ell m}$ is the underlying (identical for both population) dark matter map/distribution.

1 Standard approach

In the "standard analysis", with $a_{\ell m}$ as observables, we have as data vector simply:

$$V = \begin{pmatrix} a^{A}_{\ell m} \\ a^{B}_{\ell m} \end{pmatrix} \tag{3}$$

One would then need to compute the covariance matrix of those observable (in order to do some Fisher later on). Here are the various terms:

$$\mathrm{Cov}(a^{A}_{\ell m}, a^{A}_{\ell m}) = b_A^2\, \mathcal{C}^{M}_{\ell} + \mathcal{N}^{A}_{\ell} \tag{4}$$

$$\tag{5}$$

$$\mathrm{Cov}(a^{A}_{\ell m}, a^{B}_{\ell m}) = b_A\, b_B\, \mathcal{C}^{M}_{\ell} \tag{6}$$

$$\tag{7}$$

$$\mathrm{Cov}(a^{B}_{\ell m}, a^{B}_{\ell m}) = b_B^2\, \mathcal{C}^{M}_{\ell} + \mathcal{N}^{B}_{\ell} \tag{8}$$

$$\simeq b_B^2\, \mathcal{C}^{M}_{\ell} \tag{9}$$

The approximation in the last line corresponds to the assumption that population B has a significantly high number density of galaxies, which allows us to ignore its corresponding Poisson contribution.

NB: one can note that the correlation factor between the two $a_{\ell m}$ is the following:

$$\mathrm{Corr}(a^{A}_{\ell m}, a^{B}_{\ell m}) \equiv \frac{\mathrm{Cov}(a^{A}_{\ell m}, a^{B}_{\ell m})}{\sqrt{\mathrm{Cov}(a^{A}_{\ell m}, a^{A}_{\ell m})\mathrm{Cov}(a^{B}_{\ell m}, a^{B}_{\ell m})}} \tag{10}$$

$$= \frac{b_A\, b_B\, \mathcal{C}^{M}_{\ell}}{\sqrt{(b_A^2\, \mathcal{C}^{M}_{\ell} + \mathcal{N}^{A}_{\ell})\ (b_B^2\, \mathcal{C}^{M}_{\ell} + \mathcal{N}^{B}_{\ell})}} \tag{11}$$

$$= \frac{1}{\sqrt{(1 + \mathcal{N}^{A}_{\ell}/(b_A^2\, \mathcal{C}^{M}_{\ell}))(1 + \mathcal{N}^{B}_{\ell}/(b_B^2\, \mathcal{C}^{M}_{\ell}))}} \tag{12}$$

$$\simeq \frac{1}{\sqrt{1 + \mathcal{N}^{A}_{\ell}/(b_A^2\, \mathcal{C}^{M}_{\ell})}} \tag{13}$$

As the ratio $\mathcal{N}^{A}_{\ell}/(b_A^2\, \mathcal{C}^{M}_{\ell})$ gets closer to 0 (i.e. the ratio between the Poisson contribution to the $\mathcal{C}_\ell$ of A and its "cosmological" contribution), the correlation between the two $a^{A}_{\ell m}$ gets closer to one: you're essentially probing the same distribution.